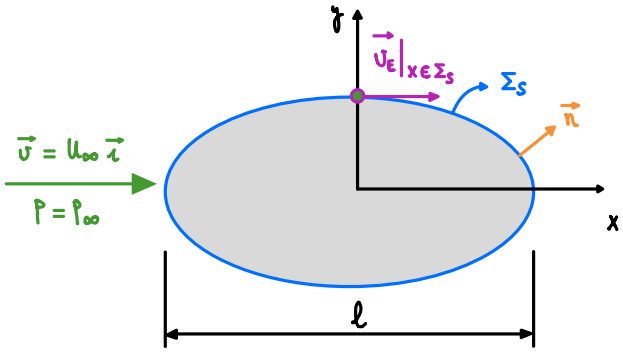
Capa Límite Laminar: Ecuaciones



Buscamos soluciones laminares, estacionarias e incompresibles.

$$\vec{v}\,' = \frac{\vec{v}}{U_\infty} \qquad \nabla'\blacksquare = \frac{1}{\ell}\nabla\blacksquare$$

$$\vec{x}\,' = \frac{\vec{x}}{\ell} \qquad \nabla'\cdot\blacksquare = \frac{1}{\ell}\nabla\cdot\blacksquare$$

$$p' = \frac{P - P_\infty}{\rho U_\infty^2} \qquad \Delta'\blacksquare = \frac{1}{\ell^2}\Delta\blacksquare$$

$$Re = \frac{U_\infty \ell}{\nu} \gg 1$$

$$P - P_\infty = \frac{1}{2}\rho\,\vec{v}\cdot\vec{v} = \frac{1}{2}\rho U_\infty^2$$

$$\nabla\cdot\vec{v} = 0$$

$$\vec{v}\,\nabla\vec{v} = -\nabla\left(\frac{P-P_\infty}{\rho}\right) + \nu\,\Delta\vec{v}$$

$$\vec{x}\in\Sigma_s : \vec{v} = \vec{0}$$

$$|\vec{x}| \to \infty : \vec{v} \to U_\infty\vec{i} + O\left(\frac{\Gamma}{|\vec{x}|}\right)$$

$$\frac{P_E - P_\infty}{\rho} \to 0 + O\left(\frac{\Gamma}{|\vec{x}|}\right)$$

$$\nabla'\cdot\vec{v}\,' = 0$$

$$\vec{v}\,'\,\nabla'\vec{v}\,' = -\nabla' p' + Re^{-1}\Delta'\vec{v}\,'$$

$$\vec{x}\,'\in\Sigma_s : \vec{v}\,' = \vec{0}$$

$$|\vec{x}\,'| \to \infty : \vec{v}\,' \to \vec{i} + (\ )$$

$$p \to 0 + (\ )$$

Solución de Euler ($\epsilon = 0$, $Re \to \infty$):

$$\left.\begin{aligned}
&\nabla\cdot\vec{u}_E = 0\\
&\vec{u}_E\,\nabla\vec{u}_E = -\nabla\left(\frac{P_E - P_\infty}{\rho}\right)\\
&\vec{x}\in\Sigma_s : \vec{u}_E\cdot\vec{n}_s = 0 + (\ )\\
&\vec{x}\to\infty : \frac{P - P_\infty}{\rho} \to 0 + (\ )
\end{aligned}\right\} \longrightarrow \vec{u}_E,\ \frac{P_E - P_\infty}{\rho}$$

EQUIVALENTE A u_E DEL EJEMPLO DE EXPANSIONES ASINTÓTICAS ACOPLADAS

La presión total es un invariante en todo el flujo $\longrightarrow$ el campo de presiones viene dado por el de presiones por la relación: $p = cte = P_t = -\frac{1}{2}\rho U_\infty^2$

Tenemos que eliminar la componente tangencial de $\vec{v}\in\Sigma_s$ que es $\neq 0$ en Euler $\longrightarrow$ HAY QUE ACTIVAR EL TÉRMINO VISCOSO CERCA DE LA PARED $\longrightarrow$ NUEVA ESCALA "δ" CERCA DE LA PARED.

$$\nabla\cdot\vec{v} = 0$$

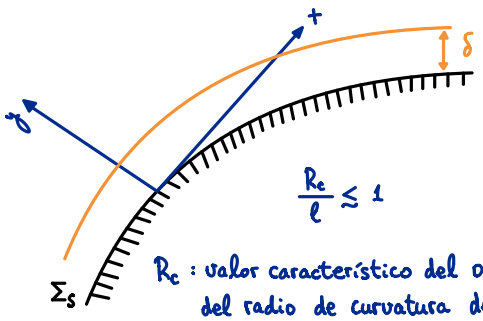
$$\underbrace{\vec{v}\,\nabla\vec{v}}_{\sim\frac{U_\infty^2}{\ell}} = -\nabla\left(\frac{P-P_\infty}{\rho}\right) + \underbrace{\nu\,\Delta\vec{v}}_{\sim\nu\frac{U_\infty}{\delta^2}}$$

Comparando órdenes de magnitud de los términos convectivo y viscoso :

$$\boxed{\frac{\delta}{\ell} \sim Re^{-1/2}}$$

FLUJO ESBELTO EN LA REGIÓN INTERIOR (CAPA LÍMITE)

$$\boxed{\frac{\Delta_y\left(\frac{P}{\rho}\right)}{\Delta_x\left(\frac{P}{\rho}\right)} \sim \left(\frac{\delta}{\ell}\right)^2 \sim Re^{-1} \ll 1}$$



$$\frac{R_c}{\ell} \lesssim 1$$

R_c : valor característico del orden del radio de curvatura de Σ_s

Importante :

$$\frac{P_e}{\rho} + \frac{U_e^2}{2} = \frac{P_\infty}{\rho} + \frac{U_\infty^2}{2} \longrightarrow \frac{\partial}{\partial x}\left(\frac{P_E}{\rho}\right) = -\frac{d}{dx}\left(\frac{U_e^2}{2}\right) = -U_e\frac{dU_e}{dx} \xrightarrow{P \sim P_E} \boxed{\frac{\partial}{\partial x}\left(\frac{P}{\rho}\right) = -U_e\frac{dU_e}{dx}}$$

Por tanto :

SOLUCIÓN INTERIOR

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y} = U_e\frac{dU_e}{dx} + \nu\frac{\partial^2 u}{\partial y^2}$$

$$0 \approx \frac{\partial}{\partial y}\left(\frac{P}{\rho}\right)$$

$y = 0$: $u = 0$; $v = v_p$ (SUCCIÓN SOPLADO)

$y \to \infty$: $u \to U_E\big|_{\vec{x}\to\Sigma_s} = U_e$

$x = x_0$: $u = u(x_0, y) = u_0(y)$

Llamando :

$$\tilde{x} = \frac{x}{\ell} \; ; \quad \tilde{y} = \frac{y}{\delta} = \frac{y}{\ell}Re^{-1/2} \; ; \quad \tilde{u} = \frac{u}{U_{ec}} \; ; \quad \tilde{v} = \frac{v}{v_c} = \frac{v}{U_{ec}}Re^{1/2} \; ; \quad \tilde{U}_e = \frac{U_e}{U_{ec}}$$

Obtenemos :

$$\frac{\partial \tilde{u}}{\partial \tilde{x}} + \frac{\partial \tilde{v}}{\partial \tilde{y}} = 0 \; ; \quad \tilde{u}\frac{\partial \tilde{u}}{\partial \tilde{x}} + \tilde{v}\frac{\partial \tilde{v}}{\partial \tilde{y}} = \tilde{U}_e\frac{d\tilde{U}_e}{d\tilde{x}} + \nu\frac{\partial^2 \tilde{u}}{\partial \tilde{y}^2}$$

$\tilde{y} = 0$: $\tilde{u} = 0$; $\tilde{v} = \tilde{v}_p$

$\tilde{y} \to \infty$: $\tilde{u} \to \tilde{U}_e$

$\tilde{x} = \tilde{x}_0$: $\tilde{u} = \tilde{u}(\tilde{x}_0, \tilde{y}) = \tilde{u}_0(\tilde{y})$

$$\longrightarrow \boxed{\begin{gathered}\tilde{u} = \tilde{u}(\tilde{x}, \tilde{y}\,;\, \tilde{U}_e, \tilde{v}_p, \tilde{x}_0, \tilde{u}_0) \\ \tilde{v} = \tilde{v}(\tilde{x}, \tilde{y}\,;\, \tilde{U}_e, \tilde{v}_p, \tilde{x}_0, \tilde{u}_0) \\ \tilde{u}, \tilde{v} = O(1)\end{gathered}}$$

INDEPENDIENTE DEL REYNOLDS

↳ PODEMOS RESOLVER LA C.L. PARA CUALQUIER REYNOLDS (∀ VELOCIDAD SUCCIÓN/SOPLADO)

Cuando deshacemos el cambio sí aparece el Reynolds : $\uparrow Re \; \downarrow \delta$

$$\delta(10^4) = \frac{\delta(10^3)}{\sqrt{10}} \approx \frac{\delta(10^3)}{3}$$